Mathematics 1502 Test number 1 Thursday, 23 September 1999 version σ

NAME_____

Instructions: Write the answers where indicated and give clear evidence of your reasoning (or points will be taken off). You may attach extra sheets with your work if it is organized enough to be helpful. Graphs should be clearly labeled. **Calculators are not permitted if they can store formulae or do symbolic mathematics (algebra & calculus).** Graphing is OK.

NOTE: The lines "KEY FORMULA OR METHOD" are provided so that if you are not going to solve the problem completely, you can show that you have some correct idea. They are not required. All answers should be as specific as possible. A "specific expression" is one you could show to someone who knows calculus, so that person could evaluate it without being shown the original problem or told anything. It should contain no expressions like "f(x)," only specific functions like "sin(x)."

SCORING - DO NOT WRITE ANSWERS ON THIS PAGE:

1 |__

2 |__

3 |__

4 |__

TOTAL_____

NAME_____

1 (10 points) Evaluate the following, if they exist. If they are divergent, state clearly why.

a) $\lim_{x \to \infty} \frac{2x^2 + x - 3}{x^2 + 3x - 4}$ = _____

b) $\lim_{x \to 1} \frac{2x^2 + x - 3}{x^2 + 3x - 4}$ = _____

c) $\lim_{x \to -4} \frac{2x^2 + x - 3}{x^2 + 3x - 4}$ = _____

d) $\sum_{n=2}^{\infty} 2^k 3^{-k+1}$ = _____

KEY FORMULA OR METHOD (optional for partial credit)_____
_____

2 (10 points) Determine whether the following converge.

a) $\int_0^2 \sqrt[3]{\frac{2+x}{2x}}\, dx$.

This integral is convergent/divergent (circle one), because _____

_____

b) $\sum_{k=2}^{\infty} \frac{1}{k(\ln(k))^3}$.

This series is convergent/divergent (circle one), because _____

_____

KEY FORMULA OR METHOD (optional for partial credit)_____
_____

NAME_____

Problems 3 and 4 are concerned with estimating the integral $\frac{3}{1+4x^2}$. No credit will be given for an accurate estimate of this integral, only for the approximation,.

3. (10 points) In this problem we use Taylor's polynomial and series.

a) Find the Taylor polynomial in powers of x up to and including x^4 for $\frac{3}{1+4x^2}$.

$p_4(x)$ = _____+_____ x + _____ x^2 + _____ x^3 + _____ x^4

b) Evaluate the integral $\int_0^{1/2} p_4(x)\,dx$.

$\int_0^{1/2} p_4(x)\,dx$ = _____

c) For what positive values of x is the Taylor series convergent for $\int_0^x \frac{3}{1+4t^2}\,dt$?

ANSWER: It converges for exactly the following values of x:_____.

KEY FORMULA OR METHOD (optional for partial credit)_____
_____

4. (10 points)

a) Evaluate the integral $\int_0^{1/2} \frac{3}{1+4x^2}\,dx$ with the trapezoid rule, n=4:

$\int_0^{1/2} \frac{3}{1+4x^2}\,dx \doteq$ _____

b) Evaluate the integral $\int_0^{1/2} \frac{3}{1+4x^2}\,dx$ with Simpson's rule, n=**2**:

$\int_0^{1/2} \frac{3}{1+4x^2}\,dx \doteq$ _____

KEY FORMULA OR METHOD (optional for partial credit)_____