

NAME\_\_\_\_\_

Instructions: Write the answers where indicated and give clear evidence of your reasoning (or points will be taken off). You may attach extra sheets with your work if it is organized enough to be helpful. Graphs should be clearly labeled. **Calculators are not permitted if they can store formulae or do symbolic mathematics (algebra & calculus).** Graphing is OK.

NOTE: The lines "KEY FORMULA OR METHOD" are provided so that if you are not going to solve the problem completely, you can show that you have some correct idea. They are not required. All answers should be as specific as possible.

**SCORING - DO NOT WRITE ANSWERS ON THIS PAGE:**

1 | \_\_\_\_\_

2 | \_\_\_\_\_

3 | \_\_\_\_\_

4 | \_\_\_\_\_

**TOTAL** \_\_\_\_\_

NAME\_\_\_\_\_

**1** (10 points) Evaluate the following, if they exist. If they are divergent, state clearly why.

a)  $\lim_{x \rightarrow \infty} \frac{\ln(x^3)}{x^2} =$ \_\_\_\_\_.

b)  $\lim_{x \rightarrow 0} \frac{\arctan(2x)}{\sin(x)} =$ \_\_\_\_\_

c)  $\sum_{n=1}^{\infty} \frac{2^{n+1}}{n!} =$ \_\_\_\_\_

Hint for c): It resembles a top-ten Taylor series, doesn't it?

KEY FORMULA OR METHOD (optional for partial credit)\_\_\_\_\_

**2** (10 points) Determine whether the following converge.

a)  $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^2 - 7}$  .

This series is convergent/divergent (circle one), because \_\_\_\_\_

b)  $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}(2k)!}{(k+5)!}$  .

This series is convergent/divergent (circle one), because \_\_\_\_\_

KEY FORMULA OR METHOD (optional for partial credit)\_\_\_\_\_

NAME \_\_\_\_\_

3. Let  $f(x) := x \cos(x) - \sin(x)$ .

a) Find the Taylor series (with  $a=0$ ) of  $f(x)$ :

$$f(x) = \underline{\hspace{4cm}}$$

b) Find the radius of convergence of the series in a):

$$r = \underline{\hspace{4cm}}$$

c) Use the Taylor series to evaluate the eleventh derivative  $\frac{d^{11} f(x)}{dx^{11}}$  when  $x=0$ :

$$f^{(11)}(0) = \underline{\hspace{4cm}}$$

KEY FORMULA OR METHOD (optional for partial credit) \_\_\_\_\_  
\_\_\_\_\_

4. Let  $f(x) = \exp(x^2-1) - 1$ . Although in this problem you are trying to estimate  $f(-1)$ , no credit will be given for realizing that  $f(-1)=0$ .

a) Find the quadratic polynomial  $P(x)$  such that  $P(1) = f(1)$ ,  $P'(1) = f'(1)$ , and  $P''(1) = f''(1)$ :

$$P(x) = \underline{\hspace{1cm}}(\underline{\hspace{1cm}})^2 + \underline{\hspace{1cm}}(\underline{\hspace{1cm}}) + \underline{\hspace{1cm}}$$

b) Evaluate  $P(-1) = \underline{\hspace{4cm}}$

c) Use Lagrange's formula to bound the error in part b):

$$| P(-1) - f(-1) | \quad \underline{\hspace{4cm}}$$

(don't just write a formula, give a specific number)