

NAME \_\_\_\_\_

Instructions: Write the answers where indicated and give clear evidence of your reasoning (or points will be taken off). You may attach extra sheets with your work if it is organized enough to be helpful. Graphs should be clearly labeled. **Calculators are not permitted if they can store formulae or do symbolic mathematics (algebra & calculus).** Graphing is OK.

NOTE: The lines "KEY FORMULA OR METHOD" are provided so that if you are not going to solve the problem completely, you can show that you have some correct idea. They are not required. All answers should be as specific as possible. A "specific expression" is one you could show to someone who knows calculus, so that person could evaluate it without being shown the original problem or told anything. It should contain no expressions like " $f(x)$ ," only specific functions like " $\sin(x)$ ."

**SCORING - DO NOT WRITE ANSWERS ON THIS PAGE:**

1 | \_\_\_\_\_2 | \_\_\_\_\_3 | \_\_\_\_\_4 | \_\_\_\_\_TOTAL \_\_\_\_\_

NAME\_\_\_\_\_

0. What is the topic of the term paper you will hand in next month?

ANSWER:\_\_\_\_\_

\_\_\_\_\_

1. (10 points) Consider the triangle with vertices

$(1,1,1)$ ,  $(1,-2,3)$ , and  $(1,0,0)$ .

a) The area of the triangle is \_\_\_\_\_

b) A vector perpendicular to the plane of the triangle is:

$\mathbf{N} =$

\_\_\_\_\_

c) A parallelepiped has one corner at the origin and edges which connect the origin to  $(1,1,1)$ ,  $(1,-2,3)$ , and  $(1,0,0)$ .

The volume of the parallelepiped is \_\_\_\_\_

KEY FORMULA OR METHOD (optional for partial credit)\_\_\_\_\_

\_\_\_\_\_

2. (10 points)

Find all solutions of the following linear systems, or show that there is no such solution:

a)

$$3x + 2y + z = 3$$

$$2x + y + z = 0$$

$$6x + 2y + 4z = 6$$

b)

$$-x + y + 2z = 3$$

$$3x - y + z = 0$$

$$-x + 3y + 4z = 6$$

KEY FORMULA OR METHOD (optional for partial credit)\_\_\_\_\_

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3. (10 points). Let

$$A := \begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix}, \quad B := \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

Calculate the following or explain why they are not defined:

a)  $\det(A) + \det(B) =$  \_\_\_\_\_

b)  $(A^{-1})^2 =$   
\_\_\_\_\_

c)  
 $B^{-1} =$   
\_\_\_\_\_

KEY FORMULA OR METHOD (optional for partial credit)\_\_\_\_\_  
\_\_\_\_\_

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4. A 3 by 3 matrix  $M$  has the following properties:

(i). If  $\mathbf{v}$  is a position vector on the  $y$ -axis, then  $M \mathbf{v}$  is the closest point to  $\mathbf{v}$  on the line  $x=0, y = z = t$ .

(ii). If  $\mathbf{v}$  lies in the  $x$ - $z$  plane, then  $M \mathbf{v}$  is rotated about the  $y$ -axis.

(iii)  $M \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$ .

a) The matrix is

$M =$   
\_\_\_\_\_

b)  $M \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \text{_____} \\ \text{_____} \\ \text{_____} \end{bmatrix}$ .

KEY FORMULA OR METHOD (optional for partial credit)\_\_\_\_\_  
\_\_\_\_\_