Mathematics 1502   Test number 4   Friday, 6 April 2001   version
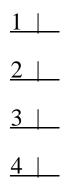
T.A. _____
NAME_____
**By signing here you acknowledge being
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Instructions: Write the answers where indicated and give clear evidence of your reasoning (or points will be taken off). You may attach extra sheets with your work if it is organized enough to be helpful. Graphs should be clearly labeled. **Calculators are not permitted if they can store formulae, do symbolic mathematics (algebra & calculus), or calculate determinants, cross products, etc..** Graphing is OK.

NOTE: The lines "KEY FORMULA OR METHOD" are provided so that if you are not going to solve the problem completely, you can show that you have some correct idea. They are not required. All answers should be as specific as possible.

Important. Think about what knd of a mathematical object a question is asking for. You will receive 0 points if your answer is not the "right kind of animal."

SCORING - DO NOT WRITE ANSWERS ON THIS PAGE:

1 |___

2 |___

3 |___

4 |___

TOTAL _____

1 (10 points). Consider the matrix

$$\mathbf{A} := \begin{bmatrix} 0 & 3 & -3 & 1 \\ 3 & 3 & -3 & 5 \\ 1 & 2 & -2 & 2 \end{bmatrix}$$

a) Find the column space of **A**. (This is the same as the image of **A**.) Describe the column space using a basis.

$\mathrm{Col}(\mathbf{A}) = \mathrm{Img}(\mathbf{A}) =$

_____

b) If the vector $\begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$ is in the column space Img(**A**), then express it as a linear combination of the columns of A. Use the smallest possible number of columns of **A**. If the vector is not in the column space, then write it as the sum of something in the column space and something orthogonal to the column space.

$\begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix} =$ _____

c) Find an orthogonal basis for the column space of **A**.

ANSWER:

_____

KEY FORMULA OR METHOD (optional for partial credit)_____

_____

210 points). Let

$$A := \begin{bmatrix} 0 & 3 & -3 & 1 \\ 3 & 3 & -3 & 5 \\ 1 & 2 & -2 & 2 \end{bmatrix}$$

It may be helpful to notice that **A** is the same as in the previous problem.

a) Fill in the blanks: The dimension of the column space of **A** is _____ and the dimension of the null space of **A** is _____

b) Find the null space of **A**. (This is the same as the kernel of **A**.) Describe the null space using a basis.

Null(**A**) = Ker(**A**) =

_____

c)

Find all solutions of **A x** = **b**, where

$$b := \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}.$$

ANSWER:

x =

_____

KEY FORMULA OR METHOD (optional for partial credit)_____

_____

3(10 points). Find the line $y = m x + b$ that best fits the data given below in the least-square sense:

x	y
0	2
1	2
3	0

ANSWER: y = _____x + _____

KEY FORMULA OR METHOD (optional for partial credit)_____

_____

4 (10 points), In Berklee, the college town designed by the astigmatic city planner Aloysius P. Berkl, the streets are all straight. There are two sets of parallel streets, aligned with the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 12 \\ 5 \end{bmatrix} \text{ and } \mathbf{v}_2 = \begin{bmatrix} 5 \\ 12 \end{bmatrix}.$$

Here the vector **i** points east and the vector **j** points north. The blocks of the streets parallel to $\mathbf{v}_1$ are 26 meters long, while the blocks parallel to $\mathbf{v}_2$ are 130 meters long

a) Find the matrix which transforms the numbers of blocks in the directions $\mathbf{v}_1$ and $\mathbf{v}_2$. into distances x (east) and y (north):

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \quad\quad\quad \end{bmatrix} \begin{bmatrix} \text{blocks parallel to } \mathbf{v}_1 \\ \text{blocks parallel to } \mathbf{v}_2 \end{bmatrix} \quad \text{(fill in blank space).}$$

b) Find the matrix which transforms position vectors $\begin{bmatrix} x \\ y \end{bmatrix}$ into blocks in the directions $\mathbf{v}_1$ and $\mathbf{v}_2$.

$$\begin{bmatrix} \text{blocks parallel to } \mathbf{v}_1 \\ \text{blocks parallel to } \mathbf{v}_2 \end{bmatrix} = \begin{bmatrix} \quad\quad\quad \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{(fill in blank space).}$$

c) If you need to lay some cable for the new bookstore at position $(x,y) = (2000,1000)$ as measured from the center of the campus, find the street directions to this position:

It is at _____ blocks parallel to $\mathbf{v}_1$ and _____ blocks parallel to $\mathbf{v}_2$ from the center of the campus. (Extra credit: Find the nearest point actually on a street to this position: _____ Assume streets have 0 thickness.)

KEY FORMULA OR METHOD (optional for partial credit)_____

_____