

NAME _____
T/A _____

Instructions: Write the answers where indicated and give clear evidence of your reasoning (or points will be taken off). You may attach extra sheets with your work if it is organized enough to be helpful. Graphs should be clearly labeled. **Calculators are not permitted if they can store formulae or do symbolic mathematics (algebra & calculus).** Graphing is OK.

NOTE: The lines "KEY FORMULA OR METHOD" are provided so that if you are not going to solve the problem completely, you can show that you have some correct idea. They are not required. All answers should be as specific as possible. A "specific expression" is one you could show to someone who knows calculus, so that person could evaluate it without being shown the original problem or told anything. It should contain no expressions like " $f(x)$," only specific functions like " $\sin(x)$."

SCORING - DO NOT WRITE ANSWERS ON THIS PAGE:

1 | _____2 | _____3 | _____4 | _____5 | _____6 | _____7 | _____8 | _____TOTAL _____

NAME_____

1 (10 points) Discuss the convergence of the following.

a) $\int_0^2 \left(\frac{2+x}{2x} \right)^{1/4} dx$.

This integral is convergent/divergent (circle one), because _____

b) $\sum_{k=2}^{\infty} \left(\frac{\ln(k)}{k-1} \right)^3$.

This series is convergent/divergent (circle one), because _____

KEY FORMULA OR METHOD (optional for partial credit)_____

2 (10 points)

a) Find the Taylor series for $f(x) := \ln(1 + 2x^2)$ (about $x=0$):

The Taylor series is:_____

b) Find the radius of convergence of the series in part a)

$r =$ _____

c) Keeping terms up to and including x^6 , estimate the error for $x=.5$, and fill in the blanks:

_____ $f(.5)$ _____

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3. (10 points) Consider the triangle in three-space with vertices at (2,2,3), (5,5,6), and (3,2,2).

a) The area of the triangle is _____

b) The formula for the plane containing the triangle is

$$_____ x + _____ y + _____ z = _____$$

and its normal vector is _____

c) If the triangle is rotated about the z-axis counterclockwise by $\pi/3$ radians, the vertices are moved to (____,____,____), (____,____,____), and (____,____,____).

KEY FORMULA OR METHOD (optional for partial credit)_____

4. (10 points).

a) Find the line which matches the following data best (in the mean-square sense):

x_k	y_k
-1	-2
0	1
1	2
2	4

ANSWER

$$y = _____ x + _____$$

Now suppose the data above are column vectors \mathbf{x} and \mathbf{y} in four-space. Find

b) the length of \mathbf{x} : _____

c) the vector parallel to \mathbf{x} which is closest to \mathbf{y} : _____

KEY FORMULA OR METHOD (optional for partial credit)_____

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5. (10 points) Consider the system of linear equations:

$$\begin{aligned}w + 2x + 3y + 4z &= 12 \\ 2w + 3x + 4y &= 12 \\ w + x + y - 4z &= 0\end{aligned}$$

a) How many solutions does the linear system have? If there are many, describe the set of solutions in a quantitative way.

ANSWER:_____

b) Find the solution(s) or explain why one cannot:

w = _____

x = _____

y = _____

z = _____

6. (10 points) Let A be the 2×2 matrix such that

$$A \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} \text{ and } A \begin{bmatrix} 0 \\ 2 \end{bmatrix} = (-1) \begin{bmatrix} 0 \\ 2 \end{bmatrix}.$$

a) The explicit form of A is:

$$A = \begin{bmatrix} _ & _ \\ _ & _ \end{bmatrix}.$$

b) Diagonalize A, in the sense that $A = U D U^{-1}$:

$$A = \begin{bmatrix} _ & _ \\ _ & _ \end{bmatrix} \begin{bmatrix} _ & 0 \\ 0 & _ \end{bmatrix} \begin{bmatrix} _ & _ \\ _ & _ \end{bmatrix}.$$

KEY FORMULA OR METHOD (optional for partial credit)_____

NAME_____

7. (10 points) In this problem it may be helpful to know that

$\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ is an eigenvector of $B := \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$.

a) Find all eigenvalues and eigenvectors of B.

ANSWER:

b) Solve the following system of differential equations, assuming that $x(0) = 1$ and $y(0) = +2$:

$$x'(t) = 2 x(t) + 2 y(t)$$

$$y'(t) = 2 x(t) - y(t)$$

$$x(t) = \text{_____}, y(t) = \text{_____}$$

KEY FORMULA OR METHOD (optional for partial credit)_____

8. (10 points) Let

$$C := \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & -2 \\ 2 & -2 & 4 \end{bmatrix}.$$

It happens that $C = 3 R + 3 I$, where R is the matrix of the reflection through a plane. (This hint might save time, if you see the point, but is not at all necessary.) Find the following, or explain why one cannot.

a) $\det C =$ _____

b) The dimension of the column space (= "range"= "image space") of C: _____

c) An orthonormal basis for the column space of C:

d) The null space (= "kernel") of C:

KEY FORMULA OR METHOD (optional for partial credit)_____
