DEAR 1512 STUDENTS,

WONDERING WHAT NEXT WEEK'S TEST WILL LOOK LIKE?

ATTACHED BELOW IS A COPY OF A TEST I GAVE LAST YEAR. DUE TO SMALL DIFFERENCES IN THE CALENDAR AND SYLLABUS, I HAVE SUPPRESSED QUESTION 4, WHICH COVERED MATERIAL WE HAVE NOT ENCOUNTERED YET. ON THE OTHER HAND, I WOULD PROBABLY ADD A PROBLEM ON POLAR COORDINATES AND/OR COMPLEX NUMBERS IN ITS PLACE.

AT THE END OF THESE THREE PROBLEMS ARE THREE VARIANTS. IN THE PAST I HAVE USUALLY GIVEN TWO VERSIONS OF A TEST, DISTRIBUTED TO ALTERNATING COLUMNS IN A CROWDED CLASSROOM.

LATER WE WILL POST MAPLE'S SOLUTIONS TO THIS TEST. _____

Mathematics 1502 Test number 1 Thursday, 23 September 1999 version

NAME _____

Instructions: Write the answers where indicated and give clear evidence of your reasoning (or points will be taken off). You may attach extra sheets with your work if it is organized enough to be helpful. Graphs should be clearly labeled. **Calculators** are not permitted if they can store formulae or do symbolic mathematics (algebra & calculus). Graphing is OK.

NOTE: The lines "KEY FORMULA OR METHOD" are provided so that if you are not going to solve the problem completely, you can show that you have some correct idea. They are not required. All answers should be as specific as possible. A "specific expression" is one you could show to someone who knows calculus, so that person could evaluate it without being shown the original problem or told anything. It should contain no expressions like "f(x)," only specific functions like "sin(x)."

SCORING - DO NOT WRITE ANSWERS ON THIS PAGE:





1 (10 points) Evaluate the following, if they exist. If they are divergent, state clearly why.

a)
$$\lim_{x \to \infty} \frac{2x^2 + x - 3}{x^2 + 3x - 4} =$$

b) $\lim_{x \to 1} \frac{2x^2 + x - 3}{x^2 + 3x - 4} =$ ______
c) $\lim_{x \to -4} \frac{2x^2 + x - 3}{x^2 + 3x - 4} =$ ______
d) $\lim_{n=2} 2^{k} 3^{-k+1} =$ ______

KEY FORMULA OR METHOD (optional for partial credit)_____

2 (10 points) Determine whether the following converge.

a) $\int_{0}^{2} \sqrt[3]{\frac{2+x}{2x}} dx$.

This integral is convergent/divergent (circle one), because _____

b)
$$\sum_{k=2}^{k=2} \frac{1}{k \left(\ln(k) \right)^3}.$$

This series is convergent/divergent (circle one), because _____

KEY FORMULA OR METHOD (optional for partial credit)

Problems 3 and 4 are concerned with estimating the integral $\frac{3}{1+4x^2}$. No credit will be given for an accurate estimate of this integral, only for the approximation,.

- 3. (10 points) In this problem we use Taylor's polynomial and series.
- a) Find the Taylor polynomial in powers of x up to and including x^4 for $\frac{3}{1+4x^2}$.

$$p_4(x) = ___+___x + ___x^2 + ___x^3 + __x^4$$

- b) Evaluate the integral $\int_{0}^{1/2} p_4(x) dx$. $\int_{0}^{1/2} p_4(x) dx =$ _____
- c) For what positive values of x is the Taylor series convergent for $\int_0^x \frac{3}{1+4t^2} dt$?

ANSWER: It converges for exactly the following values of x:______.

KEY FORMULA OR METHOD (optional for partial credit)_____

4 (10 points) (Suppressed)

ALTERNATIVE VERSIONS OF THESE PROBLEMS

1 (10 points) Evaluate the following, if they exist. If they are divergent, state clearly why.

a)
$$_{n=2} 2^{k} 5^{-k-1} =$$

b) $\lim_{x} \frac{x^{2} + 3x - 4}{2x^{2} + x - 3} =$ ______
c) $\lim_{x} \frac{x^{2} + 3x - 4}{2x^{2} + x - 3} =$ _____
c) $\lim_{x} \frac{x^{2} + 3x - 4}{2x^{2} + x - 3} =$ _____

KEY FORMULA OR METHOD (optional for partial credit)_____

2 (10 points) Determine whether the following converge.

a) $\int_{2} \sqrt[2]{\frac{3+x}{3x}} dx$.

This integral is convergent/divergent (circle one), because _____

b)
$$\lim_{k=2} \frac{1}{\sin k + /k}$$
.

This series is convergent/divergent (circle one), because _____

KEY FORMULA OR METHOD (optional for partial credit)_____

Problems 3 and 4 are concerned with estimating the integral $\frac{1}{4 + x^2}$. No credit will be given for an accurate estimate of this integral, only for the approximation,.

- 3. (10 points) In this problem we use Taylor's polynomial and series.
- a) Find the Taylor polynomial in powers of x up to and including x^4 for $\frac{1}{4+x^2}$.

p₄(x) = ____+ ____ x + _____ x² + _____ x³ + _____ x⁴
b) Evaluate the integral
$$\int_{0}^{1/2} p_4(x) dx$$
.
 $\int_{0}^{1/2} p_4(x) dx = _____$

c) For what positive values of x is the Taylor series convergent for $\int_0^x \frac{1}{4+t^2} dt$?

ANSWER: It converges for exactly the following values of x:______.

KEY FORMULA OR METHOD (optional for partial credit)_____

4 (10 points) (Suppressed)