Maple's solutions to the third test (left arrow version)

1. (10 points) Consider the triangle with vertices

(1,1,1), (1,-2,3), and (1,0,0).

```
> with(linalg);
Warning, new definition for norm
Warning, new definition for trace
```

[*BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stack, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian*]

```
> side1 := [1,1,1] - [1,-2,3];
```

$$side1 := [0, 3, -2]$$

```
> side2 := [1,1,1] - [1,0,0];
```

$$side2 := [0, 1, 1]$$

For both parts a) and b), we take the cross product. Let's do b) first, since the vector asked for is the cross product of the side vectors (times any non-zero number):

b) A vector perpendicular to the plane of the triangle is:

```
> normvec := crossprod(side1,side2);
```

$$normvec := [5, 0, 0]$$

a) The area of the triangle is

```
> (1/2)* norm(normvec);
```

$$\frac{5}{2}$$

c) A parallelepiped has one corner at the origin and edges which connect the origin to (1,1,1), (1,-2,3), and (1,0,0).

The volume of the parallelepiped is the triple product, of the three edges, as beginning from the origin, which is the same as the determinant of the 3 by 3 matrix constructed from them:

```
> det([[1,1,1],[1,-2,3],[1,0,0]]);
```

$$5$$

2. Find all solutions of the following linear systems, or show that there is no such solution:

a)

$3x + 2y + z = 3$

$2x + y + z = 0$

$6x + 2y + 4z = 6$

```
> mat1 := matrix(3,4,[3,2,1,3,2,1,1,0,6,2,4,6]);
```

$$mat1 := \begin{bmatrix} 3 & 2 & 1 & 3 \\ 2 & 1 & 1 & 0 \\ 6 & 2 & 4 & 6 \end{bmatrix}$$

```
> pivot(", 1,1);
```

$$\begin{bmatrix} 3 & 2 & 1 & 3 \\ 0 & \frac{-1}{3} & \frac{1}{3} & -2 \\ 0 & -2 & 2 & 0 \end{bmatrix}$$

```
> pivot(", 2,2);
```

$$\begin{bmatrix} 3 & 0 & 3 & -9 \\ 0 & \frac{-1}{3} & \frac{1}{3} & -2 \\ 0 & 0 & 0 & 12 \end{bmatrix}$$

Since we have a row of 0's at the bottom followed by a 12, there is no solution. Let's ask Maple directly:

```
> mat2 := matrix(3,3,[3,2,1,2,1,1,6,2,4]);
```

$$mat2 := \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$$

```
> linsolve(mat2,[3,0,6]);
```

(No solutions are given)

b)

$-x + y + 2z = 3$

$3x - y + z = 0$

$-x + 3y + 4z = 6$

```
> mat3 := matrix(3,4, [-1,1,2,3,3,-1,1,0,-1,3,4,6]);
```

$$mat3 := \begin{bmatrix} -1 & 1 & 2 & 3 \\ 3 & -1 & 1 & 0 \\ -1 & 3 & 4 & 6 \end{bmatrix}$$

```
> pivot(",1,1);
```

$$\begin{bmatrix} -1 & 1 & 2 & 3 \\ 0 & 2 & 7 & 9 \\ 0 & 2 & 2 & 3 \end{bmatrix}$$

```
> pivot(",2,2);
```

$$\begin{bmatrix} -1 & 0 & \frac{-3}{2} & \frac{-3}{2} \\ 0 & 2 & 7 & 9 \\ 0 & 0 & -5 & -6 \end{bmatrix}$$

Now we can backsolve.

```
> backsub(");
```

$$\left[\frac{-3}{10}, \frac{3}{10}, \frac{6}{5}\right]$$

3. Let

```
> Amat := matrix(2,2, [0,1,2,2]);
```

$$Amat := \begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix}$$

```
> Bmat := matrix(3,3, [0,-1,0,1,0,1,1,1,0]);
```

$$Bmat := \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

a) det(A) + det (B) =

```
> det(Amat) + det(Bmat);
```

$$-3$$

b)

```
> inverse(Amat);
```

$$\begin{bmatrix} -1 & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$$

```
> evalm(" &* ");
>
```

$$\begin{bmatrix} \frac{3}{2} & \frac{-1}{2} \\ -1 & \frac{1}{2} \end{bmatrix}$$

```
> inverse(Bmat);
```

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix}$$

4. A 3 by 3 matrix M has the following properties:

(i). If v is a position vector on the y-axis, then M v is the closest point to v on the line x=0, y = z = t.

(ii). If v lies in the x-z plane , then M v is rotated about the y-axis.

(iii). [redacted]

a) The matrix is.....

The key idea here is the basic modeling theorem. Fact (i) tells us that the second column is the position vector on the line x=0, y = z = t which is closest to (0,1,0). A little trig shows that this is

```
> col2 := matrix(3,1,[0,1/sqrt(2), 1/sqrt(2)]);
```

$$col2 := \begin{bmatrix} 0 \\ \frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} \end{bmatrix}$$

Now, what rotation matrix performs the actions in (ii) and (iii)? (This is really not part of the answer, only a step!):

```
> Rot := matrix(3,3,[4/5,0,-3/5,0,1,0,3/5,0,4/5]);
```

$$Rot := \begin{bmatrix} \frac{4}{5} & 0 & \frac{-3}{5} \\ 0 & 1 & 0 \\ \frac{3}{5} & 0 & \frac{4}{5} \end{bmatrix}$$

Let's check:

```
> evalm(Rot &* [3,0,4]);
```

$$[0, 0, 5]$$

```
> e1 := matrix(3,1,[1,0,0]): e3 := matrix(3,1,[0,0,1]):
> col1 := evalm(Rot &* e1);
```

$$col1 := \begin{bmatrix} \frac{4}{5} \\ 0 \\ \frac{3}{5} \end{bmatrix}$$

```
> col3 := evalm(Rot &* e3);
```

$$col3 := \begin{bmatrix} \frac{-3}{5} \\ 0 \\ \frac{4}{5} \end{bmatrix}$$

And the answer is...

```
> Mmat = augment(col1,col2,col3);
```

$$Mmat = \begin{bmatrix} \frac{4}{5} & 0 & \frac{-3}{5} \\ 0 & \frac{1}{2}\sqrt{2} & 0 \\ \frac{3}{5} & \frac{1}{2}\sqrt{2} & \frac{4}{5} \end{bmatrix}$$

```
>
```