Mathematics 6321A **Test 1** **date** 10 February 7, 2005

NAME:________________________________

Instructions: Work absolutely on your own, without notes or text. Your grade will be based Problem 1 and on the best two of the other three problems.

You may do, or redo, the fourth problem for homework next week.

1. (S-S 2.6 # 15.) Suppose f is a nonvanishing continuous function on the closed unit disc and holomorphic on the open unit disc. Prove that if

$|f(z)| = 1$ whenever $|z| = 1$,

then f is constant.

2. Evaluate

$$\int_\gamma \frac{z^2 - 1}{z\,(z^2 + 9)dz},$$

where $\gamma = re^{it}, 0 \leq t \leq 2\pi$, for all possible values of r, $0 < r < 3$ and $3 < r < \infty$.

3. Each of the following functions has an isolated singularity at z=0. Determine its nature. If it is a removable singularity, define f(0) so f is holomorphic there. If it is a pole, find the principal part of f. If it is an essential singularity, find the image under f of $\{z : 0 < |z| < \delta\}$ for arbitrarily small δ.

a. $\frac{\cos(2z)-1}{z}$.

b. $z \cos\left(\frac{2}{z}\right)$

c. $(\cos(2z) - 1)^{-1}$

4. Suppose that f(z) is a continuous mapping on C that preserves all distances. Is it true that f(z) is a composition of translations $z \to z + a$, rotations $z \to w\,z$ for some $|w| = 1$, and possibly complex conjugation? Prove or disprove.