Dear 6341 students,

The following problems will appear on the next Thursday. You may prepare them in advance, but you may not bring your solution or any notes to the test. Instead, you should be ready to write the answer without notes on the test during the allotted time.

In addition to these problems, you should expect one or two "solve it" problems.

1. Let U be a bounded, connected domain in $\underline{\mathbb{R}}^3$ with a boundary that is C^1, and let $G(\mathbf{x}, \mathbf{y})$ be the Green function for the domain U. Prove that for all $\mathbf{x}, \mathbf{y} \in U$,

$$0 < G(\mathbf{x}, \mathbf{y}) < \frac{1}{4\pi|\mathbf{x} - \mathbf{y}|}.$$

2. Consider the inhomogeneous Helmholtz equation:

$$(-\Delta + 1)u = f \qquad (HHE)$$

a) Find the fundamental solution for (HHE), i.e., a function $\Phi(\mathbf{x})$ such that $u := \Phi * f$ solves (HHE) on on $\underline{\mathbb{R}}^3$ for any given $f \in C_c^\infty(\underline{\mathbb{R}}^3)$.

Note: Your solution on the test should not only contain the correct expression for Φ, but also the way you obtained it. The voilà method will not be accepted for full credit.

b) Formulate a careful theorem for (HHE) similar to Theorem I on p. 23. In class be prepared to give a brief synopsis of the proof.

c) Suppose f is not assumed of compact support. Does your theorem or a reasonable extension allow a solution in the case of a bounded f such as $\sin(x_2 + x_1 x_3)$? A growing function such as $(1 + x_1^2)\cos(x_2 x_3)$? Justify your answer.