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1. (Section 3.5, Problem 13) Assume that $F(0) = 0$, and that u is a continuous integral solution of the conservation law

$$u_t + F(u)_x = 0 \quad \text{in} \quad \mathbf{R} \times (0, \infty),$$

$$u = g \quad \text{on} \quad \mathbf{R} \times \{t = 0\},$$

and that u has compact support in $\mathbf{R} \times [0, \infty)$. Prove that

$$\int_{-\infty}^{\infty} u(\cdot, t)dx = \int_{-\infty}^{\infty} gdx$$

for all $t > 0$.

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2. Assume that $f(x)$ and $xf(x) \in L^1(\mathbf{R})$. Show that

$$|\hat{f}(k) - \hat{f}(0)| \leq |k| \sup_k |\hat{f}'(k)| \leq \frac{1}{\sqrt{2\pi}} |k| \int_{-\infty}^{\infty} |xf(x)| dx.$$

Use this fact to show that if u solves the standard one–space–dimensional heat equation and $u(x,0) = g(x)$, where $g(x)$ and $xg(x) \in L^1(\mathbf{R})$, then

$$\sup_x \left| u(x,t) - \left(\int g(x) dx \right) \frac{1}{\sqrt{4\pi t}} \exp\left(-x^2/4t\right) \right| \leq \frac{cst.}{t} \int |xg(x)| dx.$$