Mathematics 6341 **Test 2** **date** 10 November 2000

NAME:_____

Instructions: These problems are to be done on a take-home basis, due Monday at the beginning of class. Write all answers where indicated (or points will be taken off). You should attach worksheets if they are likely to be helpful: Some credit may be given for a correct method, if it is clearly explained and convincing.

Note: You may refer to the texts by L.C. Evans and by Zachmanoglou and Thoe, notes taken in class by you or another student, and to the on-line undergraduate text, but to no other printed or electronic resources. Work entirely on your own.

NAME:_____

1. Consider the following initial-value problem for an function $w(x_1,t)$ of two variables:
{ EMBED Equation.DSMT36 **}**

$w(x_1,0) = 0$, $w_t(x_1,0) = g(x_1)$.

Here $-\infty < x_1, < \infty$.

a) Using the fact that

$u(x_1, x_2,t) := \cos(3\, x_2)\, w(x_1,t)$

solves the 2+1 dimensional wave equation, derive a formula for $w(x_1,t)$ of the form:

$$w(x_1,t) = \int_{\_\_\_}^{\_\_\_} \_\_\_\_\_ g(y_1)\, dy_1 .$$

(fill in the blanks).

b) Define an explicit

domain of dependence = _____

for a given x_1, $t > 0$, and an energy integral:

$$E := \int_{\_\_\_}^{\_\_\_} \_\_\_\_\_ dy_1$$

and use them to prove a uniqueness theorem, stated carefully here:

Theorem. If

then

Proof:

NAME:_____

2. Solve the following initial-value problem for u(x,y):

$u(x+u)\, u_x - y\,(y+u)\, u_y = 0,$

$u(1,y) = y^{1/2}$

ANSWER:

u(x,y) = _____

This solution is valid for the following values of x,y:

_____

2. Solve the fully nonlinear initial-value problem for u(x,y):

$u_x\, u_y = 1$

$u(2t, 2t) = 5t$

Either show that the solution is unique or give more than one solution:

u(x,y) = _____

_____

_____